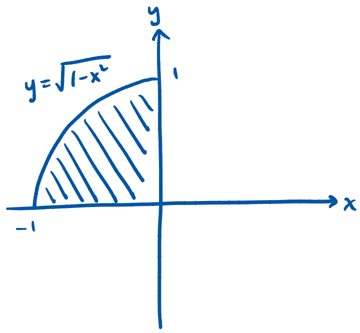
3 If we have time...

$\rho(x,y) = ky$

Example 6. A lamina occupies the part of the disk $x^2 + y^2 \leq 1$ in the second quadrant. Find its center of mass if the density at any point is proportional to its distance from the x-axis. Use Cartesian or polar coordinates. Just set up the integrals, do not evaluate.



Polar: $\frac{\pi}{2} \leq \theta \leq \pi$

$0 \leq r \leq 1$

$$m = \iint_D ky \, dA = \int_{-1}^{0} \int_{0}^{\sqrt{1-x^2}} ky \, dy \, dx \quad \text{(Cartesian)}$$

$$= \int_{\frac{\pi}{2}}^{\pi} \int_{0}^{1} kr\sin\theta \cdot r \, dr \, d\theta \quad \text{(polar)}$$

$$M_x = \iint_D y(ky) \, dA = \int_{-1}^{0} \int_{0}^{\sqrt{1-x^2}} ky^2 \, dy \, dx$$

$$= \int_{\frac{\pi}{2}}^{\pi} \int_{0}^{1} kr^2 \sin^2\theta \cdot r \, dr \, d\theta$$

$$M_y = \iint_D x(ky) \, dA = \int_{-1}^{0} \int_{0}^{\sqrt{1-x^2}} kxy \, dy \, dx$$

$$= \int_{\frac{\pi}{2}}^{\pi} \int_{0}^{1} kr^2 \cos\theta \sin\theta \cdot r \, dr \, d\theta$$

$$\Rightarrow \quad \bar{x} = \frac{M_y}{m} \qquad \bar{y} = \frac{M_x}{m}$$